# Mathematical to Predict the Compressive Strength of Sugar Cane Bagasse Ash Cement Concrete

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**Abstract**

This research work focused on the development of mathematical model for the effective replacement of Portland cement by sugar cane bagasse in mortar and concrete. The model techniques used here is Scheffe's Simplex Design. A total of ninety (90) cubes were cast, consisting of three cubes per mix ratio and for a total of thirty (30) mix ratios. The first fifteen (15) mixes were used to develop the models, while the other fifteen were used to validate the model. The mathematical model results compared favourably with the experimental data and the predictions from the model were tested with statistical student's T- test and found to be adequate at 95% confidence level. The optimum compressive strength of the blended concrete at twenty-eight (28) days was found to be 29.48Nmm2. This strength corresponded to a mix ratio of 0.55:0.9:0.10:2.8:3.2 for water: cement: sugar cane bagasse: sand: granite respectively. The model derived in this research can be used to predict mix ratios for any desired strength of Sugar Cane Bagasse ash-cement concrete and vice versa.

KEYWORDS: Sugar Cane Bagasse Ash, cement, compressive strength, mix ratio, Scheffe's simplex model, Experimental Data.



## 1.0 INTRODUCTION

The basic components of concrete are water, cement, coarse aggregate, and fine aggregate (Neville, 2011). Various chemical and mineral admixtures as well as supplementary cementitious materials (in this case sugar cane baggase ash) can be added. The proportions of these components affect properties of concrete. Such properties are shear modulus, elastic modulus, compressive strength, setting time, durability, workability, creep, shrinkage. Application of optimization principles in concrete produces an optimum concrete mix. Optimum mix being a mix with the required properties (which can be any of the above mentioned properties) and performance at a minimum price (Osadebe and Ibearugbulem 2009).

Cement is the major component of concrete used by construction industries in Nigeria. It is used in the production of concrete, mortar and sandcrete blocks which are required for the construction of buildings, dams and bridges (Anya, 2015). Most popularly used type of cement in Nigeria is Ordinary Portland Cement whose price is on the increase due to inflation, and major changes in the cement sector of Nigeria discourage Nigerians from embarking on building housing unit or large multi-storey structure.

Effort at producing low cost rural housing has been minimal. Development of supplementary cementitious material is a major step in reducing cost of producing concrete, mortar and sandcrete blocks in building construction. Also, a lot of effort has been put into the use of industrial and agro waste in more effective ways. The use of sugar cane baggase ash (SCBA), a waste from sugar cane industry reduces cost of production of concrete. These will provide a cheap, safe and effective management of sugar cane baggaseash (SCBA) as waste (Okoroafor, 2012).

However, addition of sugar cane baggase ash (SCBA) as stated before increases the component of concrete from four to five. This makes the orthodox method of mix design, which is used in predicting the properties of concrete such as compressive strength more tedious. The problem of identifying optimum concrete mix becomes very complicated and extremely complex. This is in agreement with the statement credited to Ippei et al (**2000**), which stated thus: “this proportion problem is classified as a multi criteria optimization problem and it is of vital importance to formulate a way to solve the multi criteria optimization problem. Using the orthodox method of developing mix designs will require carrying out several trials on various mix proportion in the laboratories making even more difficult to identify optimum concrete mix”. With the development of a mathematical model that will predict optimum concrete mix values of Compressive Strength of concrete and other desired properties of concrete, it becomes easier to identify an optimum concrete mix (Ezeh and Ibearugbulem (2009, 2010). Development of a mathematical model will reduce the requirement of large number of trials and makes the accommodation of extra components of concrete apart from the basic four components easier.

Mixture models have been applied in many real life applications to solve problems in such areas as in pharmacy, food industry, agriculture and engineering. Piepel and Redgate (1998) applied mixture experiment analysis to determining oxide compositions in cement clinker. Ezeh et al (2010) developed a model for the optimization of aggregate composition of laterite/sand hollow block using Scheffe’s simplex method. Mama and Osadebe (2011) developed models, one based on Scheffe’s simplex lattice and the other on Osadebe’s model, for predicting the compressive strength of sandcrete blocks using alluvial deposit. Osadebe’s model was also used by Anyaogu et al (2013) to predict the compressive strength of Pulverized Fuel Ash (PFA) – Cement concrete.

Some other works on mixture experiments include:

- Simon (2003) who developed models for concrete mixture optimization. Models were developed for many responses such as compressive strength, 1-day strength, slump and 42-day charge passed for concrete made using water, cement, silica fume, high range water-reducing admixture, coarse aggregate and fine aggregate.
- Obam (2009) – a model for optimizing shear modulus of Rice husk ash concrete.
- Onwuka et al (2011) – model for prediction of concrete mix ratios using modified regression theory.
- Osadebe and Ibearugbulem (2009) – Simplex lattice model for optimizing compressive strength of periwinkle shell-granite concrete
- Ezeh and Ibearugbulem (2009, 2010) – models for optimizing compressive strength of recycled concrete and river stone aggregate concrete respectively
- Akalin et al (2008). – Optimized chemical admixture for concrete on mortar performance tests.

## 2.0 MATERIALS AND METHODS

### 2.1 Materials

#### 2.1.1 Aggregates

The aggregates used in this research work were fine aggregate and Coarse aggregates. The fine aggregate was obtained from a flowing river (Otamiri River) purchased from mining site inside Federal University Owerri, Imo State. It was sun-dried for seven days inside the laboratory before usage. The aggregates used were free from deleterious matters. The maximum diameter of sand used was 5mm. The physical and mechanical characterization tests were performed on the sand; the values of 1564kg/m$^3$, 2.65, 1.53, 2.0 for average bulk density, specific gravity, coefficient of curvature (Cc) and uniformity (Cu) were obtained respectively. The coarse aggregate was obtained from dealers in Owerri in Imo State; the maximum size of the coarse aggregate was 19.5mm and the compacted

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bulk density of the coarse aggregate is 1615kg/m$^3$ and the non-compacted bulk density is 1400kg/m$^3$

**2.1.2 Water**

Water used for this research work was obtained from a borehole within the premises of Federal University of Technology, Owerri, Imo State. The water is potable and conforming to the standard of BS EN 1008: (2002). Since it meets the standard for drinking, it is also good for making concrete and curing concrete.

**2.1.3 Cement**

Cement can be defined as a product of calcareous (lime) and argillaceous (clay) materials which when mixed with water forms a paste and binds the inert materials like sand, gravel and crushed stones (Bhavikatti, 2001). According to BS 5328: Part 1:1997 "cement is a hydraulic binder that sets and hardens by chemical interaction with water and is capable of doing so under water".

Dangote brand of ordinary Portland cement which conforms to the requirements of BS EN 197 1:2000 was obtained from dealer in Owerri and use for all the work.

**2.2 METHODS**

**2.1 Scheffe's Optimization Model**

In this work, Henry Scheffe's optimization was used to predict possible mix proportions of concrete components that will produce a desired strength by the aid of a computer programme. Achieving a desired compressive strength of concrete is dependent to a large extent, on the adequate proportioning of the components of the concrete. In Scheffe's work, the desired property of the various mix ratios, depended on the proportion of the components present but not on the quality of mixture.

Therefore, if a mixture has a total of q components/ ingredients of the $i^{th}$ component of the mixture such that

$$X_i \geq 0 \ (i = 1, 2, 3, .., q) \qquad (3.1)$$

are interacting in a mixture. In other words, a mixture comprising of q components can be analyzed using a (q –1) space

and assuming the mixture to be a unit quantity, then the sum of all proportions of the component must be unity. That is,

$$X_1 + X_2 + X_3 + \cdots + X_{q-1} + X_q = 1 \qquad (3.2)$$

This implies that

$$\sum_{i=1}^{q} X_i = 1 \qquad (3.3)$$

Combining Eqn (3.1) and (3.3)

It implies that $0 =\leq X_i \leq 1 (3.4)$

The factor space therefore is a regular (q –1) dimensional simplex.

**3.6.1.1 Scheffe's Simplex Lattice**

A factor space is a one-dimensional (a line), a two-dimensional (a plane), a three – dimensional (a tetrahedron) or any other imaginary space where mixture component interacts. The boundary with which the mixture components interact is defined by the space.

Scheffe (1958) stated that (q–1) space would be used to define the boundary where q components

For instance:

For a mixture comprising two components i.e. q = 2, a line will be used to analyze the interaction components. Thus, it is a one-dimensional space.

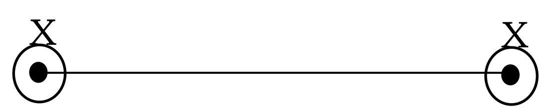


**Fig 3.1:** Two components in a one-dimensional space

a. A mixture comprising three components, i.e. q = 3, triangular simplex lattice is used in its analysis.

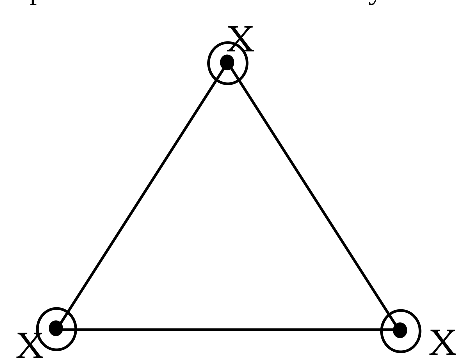

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**Fig 3.2:** Three components in a two – dimensional space

b. A mixture comprising for components, i.e. $q = 4$, a tetrahedron simplex lattice is used in its analysis.

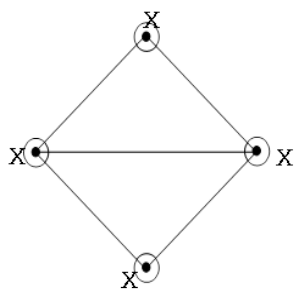


**Fig 3.3:** Four components in a three – dimensional space



**3.6.1.2 Interaction of Components in Scheffe's Factor Space**

The components of a mixture are always interacting with each other within the factor space. Three regions exist in the factor space. These regions are the vertices, borderlines, inside body space. Pure components of the mixture exist of the vertices of the factor spaces. The border line can be a line for one-dimensional or two – dimensional factor space. It can also be both lines and plane for a three – dimensional, four – dimensional, etc. factor spaces. Two components of a mixture exist at any point on the plane border, which depends on how many vertices that defined the plane border. All the component of a mixture exists right inside the body of the space.

Also, at any point in the factor space, the total quantity of the Pseudo components must be equal to one. A two – dimensional factor space will be used to clarify the interaction components. Fig 3.4: Shows a seven points on the two – dimensional factor space.

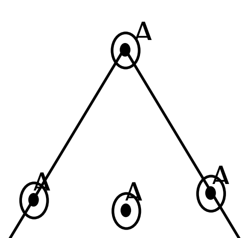


**Fig3.4:** A Two – Dimensional Space Factor

The three points, $A_1$, $A_2$ and $A_3$ are on the vertices. Three points $A_{12}$, $A_{13}$ and $A_{23}$ are on the border of space. One remaining of $A^{123}$ is right inside the body of the space.

$A_1$, $A_2$ and $A_3$ are called principal co-ordinates, only one pure component exists at any of these principal coordinates, the total quantity of the Pseudo components of these coordinates is equal to one. The other components outside these coordinates are all zero. For instance, at coordinate $A_1$, only A1 exists and the quantity of its Pseudo component is equal to one. The other components are equal to zero.

$A_{12}$, $A_{13}$ and $A_{23}$ are point or coordinates where binary mixtures occur.

At these points only two components exist and the rest do not. For instance, at point $A_{12}$, components of $A_1$ and $A_2$ exit. The total quantity of Pseudo components of $A_1$ and $A_2$ at that point is equal to one, while component $A_3$ is equal to zero at that point.

If $A_{12}$ is midway, then the component of $A_1$ is equal to half and that of $A_2$ is equal to half, while $A_3$ is equal to zero at that point. At any point inside the space, all the three components $A_1$, $A_2$ and $A_3$ exist. The total quantity of the Pseudo component is still equal to one. Consequently, if a point $A_{123}$ is exactly at the centroid of the space, the Pseudo component of $A_1$ is equal to those of $A_{2,}$ and $A_2$ and is equal to one – third $(^1/_3)$

**3.6.1.3 Five Components Factor Space**

This research work is dealing with a five component concrete mixture. The components that form the concrete mixture are water/cement (w/c) ratio, cement, sugar cane baggasse ash, , river sand and granite.

The number of components, q is equal to five. The space to be used in the analysis will be q–1, which is equal to four – dimensional factor space. A four-dimensional factor space is an imaginary dimension space.

The imaginary space used is shown in figure 3.5

below

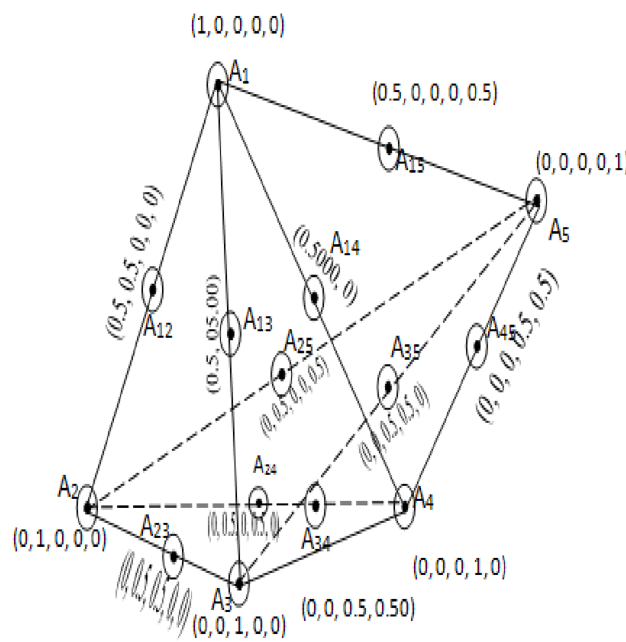


Fig.3.5 shows 15 points on the five – dimensional factor space. The properties of the Pseudo components of the five component mixture is shown in Table 3.1.



**Table 3.1**: Proportions of the Pseudo components

| Points on Factor Space | Pseudo Components |
|---|---|
| $A_1$ | (1, 0, 0, 0, 0, ) |
| $A_2$ | (0, 1, 0, 0, 0, ) |
| $A_3$ | (0, 0, 1, 0, 0, ) |
| $A_4$ | (0, 0, 0, 1, 0, ) |
| $A_5$ | (0, 0, 0, 0, 1, ) |
| $A_{12}$ | (0.5, 0.5, 0, 0, 0, ) |
| $A_{13}$ | (0.5, 0, 0.5, 0, 0, ) |
| $A_{14}$ | (0.5, 0, 0, 0.5, 0, ) |
| $A_{15}$ | (0.5, 0, 0, 0, 0.5, ) |
| $A_{23}$ | (0, 0.5, 0.5, 0, 0, ) |
| $A_{24}$ | (0, 0.5, 0, 0.5, 0, ) |
| $A_{25}$ | (0, 0.5, 0, 0, 0.5, ) |
| $A_{34}$ | (0, 0, 0.5, 0, 0.5, ) |
| $A_{45}$ | (0, 0, 0, 0.5, 0.5, ) |

### 3.6.1.4 Relationship between the Pseudo and Actual Components

In Scheffe's mixture design, the Pseudo components have relationship with the actual component. This means that the actual component can be derived from the Pseudo components and vice versa. According to Scheffe, Pseudo components were designated as X and the actual components were designated as

S. Hence the relationship between X and S as expressed by Scheffe is given in Eqn (3.5).

$$S = A * X \qquad (3.5)$$

where A is the coefficient of the relationship Eqn (3.5 ) can thus be transformed to Eqn (3.6) as

$$X = A^{-1} * S \qquad (3.6)$$

Let $A^{-1}$ = B, hence, Eqn (3.6) becomes

$$X = B * S \qquad (3.7)$$

Eqn (3.5) will be used to determine actual component of the mixture when the Pseudo components are known, while Eqns (3.6) and (3.7) will be used to determine the Pseudo components of the mixture when the actual components are known.

The six components are: Water, Cement, Sawdust ash, Palm bunch ash, Sand and Granite.

Let $S_1$ = Water; $S_2$ = Cement; $S_3$ = SCBA; $S_4$ = Sand and $S_5$ = Granite.

Then, in keeping with the principle of absolute volume

$$S_1 + S_2 + S_3 + S_4 + S_5 += S \qquad (3.8)$$

Or

$$\frac{S_1}{S} + \frac{S_2}{S} + \frac{S_3}{S} + \frac{S_4}{S} + \frac{S_5}{S} = 1 \qquad (3.9)$$

where $\frac{S_i}{S}$ is the proportion of the $ith$ constituent component of the considered concrete mix.

$$Let\,\frac{S_i}{S} = Z_i, where\ i = 1, 2, 3, 4, 5 \quad (3.10)$$

Substituting Eqn (3.10) into Eqn (3.9), we have

$$Z_1 + Z_2 + Z + Z_4 + Z_5 = 1 \quad (3.11)$$

According to Henry Scheffe's simplex lattice, the mix ratio drawn in a imaginary space will give a 21 points on the five – dimensional factor spaces.

Let Pseudo component of the mixture at a given point $A_{jk}$ on the factor space be $K_{ijk}$. The point $A_{jk}$ is an arbitrary point on the factor space and $K_{ijk}$ is the arbitrary quantities of all the Pseudo components.

The proportion of the Pseudo component of the six component mixture is given in Table 3.1. The starting set of actual components S and Pseudo Components X used in this research is shown in Table 3.2.

**Table 3.2:** Actual and Pseudo components

| N | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | Response | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.6 | 0.95 | 0.05 | 2 | 4 | $Y_1$ | 1 | 0 | 0 | 0 | 0 |
| 2 | 0.55 | 0.90 | 0.10 | 2.8 | 3.2 | $Y_2$ | 0 | 1 | 0 | 0 | 0 |
| 3 | 0.56 | 0.85 | 0.15 | 2.6 | 3.4 | $Y_3$ | 0 | 0 | 1 | 0 | 0 |
| 4 | 0.57 | 0.80 | 0.20 | 2.4 | 3.6 | $Y_4$ | 0 | 0 | 0 | 1 | 0 |
| 5 | 0.58 | 0.75 | 0.25 | 2.2 | 3.8 | $Y_5$ | 0 | 0 | 0 | 0 | 1 |

where N = any point on the factor space

Y = response

Expanding Eqn (3.5) given Eqn (3.12)

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \quad (3.12)$$

Substituting the values in Table 3.2 into Eqn(3.12) gives point N = 1

$$\begin{bmatrix}0.6\\0.95\\0.05\\2\\4\end{bmatrix} = \begin{bmatrix}a_{11} & a_{12} & a_{13} & a_{14} & a_{15}\\a_{21} & a_{22} & a_{23} & a_{24} & a_{25}\\a_{31} & a_{32} & a_{33} & a_{34} & a_{35}\\a_{41} & a_{42} & a_{43} & a_{44} & a_{45}\\a_{51} & a_{52} & a_{53} & a_{54} & a_{55}\end{bmatrix}\begin{bmatrix}1\\0\\0\\0\\0\end{bmatrix} \quad (3.13)$$

Solving Eqn (3.13), the followings were obtained

$a_{11} = 0.60$
$a_{21} = 0.95$
$a_{31} = 0.05$
$a_{41} = 2$
$a_{51} = 4$

Point N = 2

$$\begin{bmatrix}0.55\\0.90\\0.10\\2.8\\3.2\end{bmatrix} = \begin{bmatrix}a_{11} & a_{12} & a_{13} & a_{14} & a_{15}\\a_{21} & a_{22} & a_{23} & a_{24} & a_{25}\\a_{31} & a_{32} & a_{33} & a_{34} & a_{35}\\a_{41} & a_{42} & a_{43} & a_{44} & a_{45}\\a_{51} & a_{52} & a_{53} & a_{54} & a_{55}\end{bmatrix}\begin{bmatrix}0\\1\\0\\0\\0\end{bmatrix} \quad (3.14)$$

$a_{12} = 0.55$
$a_{22} = 0.9$
$a_{32} = 0.10$
$a_{42} = 2.8$
$a_{52} = 3.2$

Solving Eqn (3.14), the followings were obtained

Point N = 3

$$\begin{bmatrix}0.56\\0.85\\0.15\\2.6\\3.4\end{bmatrix} = \begin{bmatrix}a_{11} & a_{12} & a_{13} & a_{14} & a_{15}\\a_{21} & a_{22} & a_{23} & a_{24} & a_{25}\\a_{31} & a_{32} & a_{33} & a_{34} & a_{35}\\a_{41} & a_{42} & a_{43} & a_{44} & a_{45}\\a_{51} & a_{52} & a_{53} & a_{54} & a_{55}\end{bmatrix}\begin{bmatrix}0\\0\\1\\0\\0\end{bmatrix} \quad (3.15)$$

Solving Eqn (3.15), the followings were obtained

$a_{13} = 0.57$
$a_{23} = 0.85$
$a_{33} = 0.15$
$a_{43} = 2.6$
$a_{54} = 3.4$

Point N = 4

$$\begin{bmatrix}0.57\\0.8\\0.02\\2.4\\3.6\end{bmatrix} = \begin{bmatrix}a_{11} & a_{12} & a_{13} & a_{14} & a_{15}\\a_{21} & a_{22} & a_{23} & a_{24} & a_{25}\\a_{31} & a_{32} & a_{33} & a_{34} & a_{35}\\a_{41} & a_{42} & a_{43} & a_{44} & a_{45}\\a_{51} & a_{52} & a_{53} & a_{54} & a_{55}\end{bmatrix}\begin{bmatrix}0\\0\\0\\1\\0\end{bmatrix} \quad (3.16)$$

Solving Eqn (3.16), the followings were obtained

$a_{14} = 0.57$
$a_{24} = 0.8$
$a_{34} = 0.02$
$a_{44} = 2.4$
$a_{54} = 3.6$

Point N = 5

$$\begin{bmatrix}0.58\\0.75\\0.25\\2.2\\3.8\end{bmatrix} = \begin{bmatrix}a_{11} & a_{12} & a_{13} & a_{14} & a_{15}\\a_{21} & a_{22} & a_{23} & a_{24} & a_{25}\\a_{31} & a_{32} & a_{33} & a_{34} & a_{35}\\a_{41} & a_{42} & a_{43} & a_{44} & a_{45}\\a_{51} & a_{52} & a_{53} & a_{54} & a_{55}\end{bmatrix}\begin{bmatrix}0\\0\\0\\0\\1\end{bmatrix} \quad (3.17)$$

Solving Eqn (3.17), yields

$$a_{15} = 0.58$$
$$a_{25} = 0.75$$
$$a_{35} = 0.25$$
$$a_{45} = 2.2$$
$$a_{55} = 3.8$$

Assembling the coefficients of matrix A, gives

$$[A] = \begin{bmatrix} 0.60 & 0.55 & 0.56 & 0.57 & 0.58 \\ 0.95 & 0.90 & 0.85 & 0.80 & 0.75 \\ 0.05 & 0.10 & 0.15 & 0.20 & 0.25 \\ 2 & 2.8 & 2.60 & 2.40 & 2.20 \\ 4 & 3.2 & 3.40 & 3.60 & 3.80 \end{bmatrix} \quad (3.18)$$

Recall Eqn (3.6)

$$B = A^{-1} = \begin{bmatrix} 0.00 & -1.50 & -6.50 & -.125 & 0\ .5 \\ 7.223+15 & 5.946+15 & 6.235+15 & -1.43+15 & -1.86+15 \\ -6.320+15 & -9.707+15 & -9.959+15 & 1.99+15 & 2.378+15 \\ -.932+15 & 1.574+15 & 1.2134+15 & 2.823+15 & 8.240+14 \\ 8.125+15 & 2.186+15 & 2.511+15 & -8.1546+14 & -1.34+15 \end{bmatrix} \quad (3.19)$$

**3.6.1.5 Determination of Actual Components of the Binary Mixture**

The actual components of the binary mixture (as represented by points N = 12 to N = 45) are determined by multiplying matrix [A] with values of matrix [X],

[S] = [A] * [X] (3.20)

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \end{bmatrix} = \begin{bmatrix} 0.60 & 0.55 & 0.56 & 0.57 & 0.58 \\ 0.95 & 0.90 & 0.85 & 0.80 & 0.75 \\ 0.05 & 0.10 & 0.15 & 0.20 & 0.25 \\ 2 & 2.8 & 2.60 & 2.40 & 2.20 \\ 4 & 3.2 & 3.40 & 3.60 & 3.80 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix} \quad (3.21)$$

Substituting the values of Pseudo components at N = 12 into Eqn (3.21)

For N = 12

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \end{bmatrix} = [A] \begin{bmatrix} 0.5 \\ 0.5 \\ 0.0 \\ 0.0 \\ 0.0 \end{bmatrix} \quad (3.22)$$

Solving Eqn (3.22), yields

$$S_1 = 0.575$$
$$S_2 = 0.925$$
$$S_3 = 0.075$$
$$S_4 = 2.4$$
$$S_5 = 3.6$$

For point 13, where N = 13

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \end{bmatrix} = [A] * \begin{bmatrix} 0.5 \\ 0 \\ 0.5 \\ 0 \\ 0 \end{bmatrix} \quad (3.23)$$

Solving Eqn (3.23), yields

$$S_1 = 0.58$$
$$S_2 = 0.90$$
$$S_3 = 0.10$$
$$S_4 = 2.3$$
$$S_5 = 3.7$$

For point 14, where N = 14

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \end{bmatrix} = [A] \begin{bmatrix} 0.5 \\ 0.0 \\ 0.0 \\ 0.5 \\ 0 \end{bmatrix} \quad (3.24)$$

Solving Eqn (3.24), yields

$S_1 = 0.585$
$S_2 = 0.875$
$S_3 = 0.125$
$S_4 = 2.2$
$S_5 = 3.8$

For point 15, where N = 15

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \end{bmatrix} = [A] \begin{bmatrix} 0.5 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.5 \end{bmatrix} (3.25)$$

Solving Eqn (3.25), yields

$S_1 = 0.59$
$S_2 = 0.85$
$S_3 = 0.15$
$S_4 = 2.1$
$s_5 = 3.9$

For point 23, where N = 23

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ s_5 \end{bmatrix} = [A] \begin{bmatrix} 0.0 \\ 0.5 \\ 0.5 \\ 0.0 \\ 0.0 \end{bmatrix} (3.26)$$

Solving Eqn (3.26), yields

$S_1 = 0.555$
$S_2 = 0.875$
$S_3 = 0.125$
$S_4 = 2.7$
$s_5 = 3.3$

For point 24, where N = 24

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ s_5 \end{bmatrix} = [A] \begin{bmatrix} 0.0 \\ 0.5 \\ 0.0 \\ 0.5 \\ 0 \end{bmatrix} (3.27)$$

Solving Eqn (3.27), yields

$S_1 = 0.56$
$S_2 = 0.85$
$S_3 = 0.15$
$S_4 = 2.6$
$s_5 = 3.4$

For point 25 (where N = 25)

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ s_5 \end{bmatrix} = [A] \begin{bmatrix} 0.0 \\ 0.5 \\ 0.0 \\ 0.0 \\ 0.5 \end{bmatrix} (3.28)$$

Solving Eqn (3.29), yields

$S_1 = 0.565$
$S_2 = 0.825$
$S_3 = 0.175$
$S_4 = 2.5$
$s_5 = 3.5$

For point 34, where N = 34

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ s_5 \end{bmatrix} = [A] \begin{bmatrix} 0.0 \\ 0.0 \\ 0.5 \\ 0.5 \\ 0 \end{bmatrix} (3.29)$$

Solving Eqn (3.29), yields

$S_1 = 0.565$
$S_2 = 0.825$
$S_3 = 0.175$
$S_4 = 2.5$
$s_5 = 3.5$

For point 35, where N = 35

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ s_5 \end{bmatrix} = [A] \begin{bmatrix} 0.0 \\ 0.0 \\ 0.5 \\ 0.0 \\ 0.5 \end{bmatrix} (3.30)$$

Solving Eqn (3.30), yields

$S_1 = 0.57$
$S_2 = 0.8$
$S_3 = 0.2$
$S_4 = 2.4$
$s_5 = 3.6$

For point 45 (where N = 45)

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ s_5 \end{bmatrix} = [A] * \begin{bmatrix} 0.0 \\ 0.0 \\ 0.0 \\ 0.5 \\ 0.5 \end{bmatrix} \quad (3.31)$$

Solving Eqn (3.31), yields

$$S_1 = 0.575$$
$$S_2 = 0.775$$
$$S_3 = 0.225$$
$$S_4 = 2.3$$
$$s_5 = 3.7$$

The Pseudo components and the corresponding actual components at different points on the factor space are shown in Table 3.3.

**Table 3.3**: Values of Actual and Pseudo components for trial mixes

| | Values of Actual Components | | | | | | Values of Pseudo Components | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| N | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | Response | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ |
| 1 | 0.60 | 0.95 | 0.05 | 2 | 4 | $Y_1$ | 1 | 0 | 0 | 0 | 0 |
| 2 | 0.55 | 0.90 | 0.10 | 2.8 | 3.2 | $Y_2$ | 0 | 1 | 0 | 0 | 0 |
| 3 | 0.56 | 0.85 | 0.15 | 2.6 | 3.4 | $Y_3$ | 0 | 0 | 1 | 0 | 0 |
| 4 | 0.57 | 0.80 | 0.20 | 2.4 | 3.6 | $Y_4$ | 0 | 0 | 0 | 1 | 0 |
| 5 | 0.58 | 0.75 | 0.25 | 2.2 | 3.8 | $Y_5$ | 0 | 0 | 0 | 0 | 1 |
| 12 | 0.575 | 0.925 | 0.075 | 2.4 | 3.6 | $Y_{12}$ | 0.5 | 0.5 | 0 | 0 | 0 |
| 13 | 0.58 | 0.90 | 0.10 | 2.3 | 3.7 | $Y_{13}$ | 0.5 | 0 | 0.5 | 0 | 0 |
| 14 | 0.585 | 0.875 | 0.125 | 2.2 | 3.8 | $Y_{14}$ | 0.5 | 0 | 0 | 0.5 | 0 |
| 15 | 0.59 | 0.85 | 0.15 | 2.1 | 3.9 | $Y_{15}$ | 0.5 | 0 | 0 | 0 | 0.5 |
| 23 | 0.555 | 0.875 | 0.125 | 2.7 | 3.3 | $Y_{23}$ | 0 | 0.5 | 0.5 | 0 | 0 |
| 24 | 0.56 | 0.85 | 0.15 | 2.6 | 3.4 | $Y_{24}$ | 0 | 0.5 | 0 | 0.5 | 0 |
| 25 | 0.565 | 0.825 | 0.125 | 2.5 | 3.5 | $Y_{25}$ | 0 | 0.5 | 0 | 0 | 0.5 |
| 34 | 0.565 | 0.825 | 0.125 | 2.5 | 3.5 | $Y_{34}$ | 0 | 0 | 0.5 | 0.5 | 0 |
| 35 | 0.570 | 0.80 | 0.20 | 2.4 | 3.6 | $Y_{35}$ | 0 | 0 | 0.5 | 0 | 0.5 |
| 45 | 0.575 | 0.775 | 0.225 | 2.3 | 3.7 | $Y_{45}$ | 0 | 0 | 0 | 0.5 | 0.5 |

**Table 3.4: Mass of Constituents of Concrete of trial Mixes (kg)**

| N | Water | Cement | SCBA | Sand | Granite |
|---|---|---|---|---|---|
| 1 | 2.175 | 3.63 | 0.192 | 7.65 | 15.27 |
| 2 | 1.89 | 3.435 | 0.375 | 10.695 | 12.225 |
| 3 | 1.815 | 3.24 | 0.57 | 9.93 | 12.99 |
| 4 | 1.74 | 3.06 | 0.765 | 9.165 | 13.74 |
| 5 | 1.665 | 1.785 | 0.96 | 8.40 | 14.505 |
| 12 | 2.025 | 3.525 | 0.285 | 9.165 | 13.74 |
| 13 | 1.995 | 3.435 | 0.39 | 8.79 | 14.13 |
| 14 | 1.95 | 3.345 | 0.48 | 8.4 | 14.505 |

| | | | | | |
|---|---|---|---|---|---|
| 15 | 1.905 | 3.24 | 0.57 | 6.025 | 14.895 |
| 23 | 1.86 | 3.345 | 0.48 | 10.305 | 12.60 |
| 24 | 1.815 | 3.24 | 0.57 | 9.93 | 12.99 |
| 25 | 1.785 | 3.15 | 0.675 | 9.54 | 13.365 |
| 34 | 1.785 | 3.15 | 0.675 | 9.54 | 13.365 |
| 35 | 1.74 | 3.06 | 0.765 | 9.165 | 13.74 |
| 45 | 1.695 | 2.955 | 0.855 | 8.79 | 14.13 |

**Responses**

Responses according to Simon (2003) refer to any measureable plastic or hardened properties of concrete. These properties include compressive strength, flexural strength, elastic modulus; shear modulus etc. cost can also be a response. The specified properties are called the responses or dependent variables, $Y_i$, which are the performance criteria for optimizing sought is the compressive strength of Sugar cane Baggasse Ash – cement concrete. The response is presented using a polynomial function of Pseudo components of the mixture.

$$Y = b_o + \sum b_i X_i + \sum b_{ij} X_i X_j + \sum b_{ijk} X_i X_j X_k + \cdots + e \qquad (3.32a)$$

Where

$b_i$, $b_{ij}$, and $b_{ijk}$ are constants; $X_i$, $X_j$ and $X_k$ are Pseudo components; and e is the random error term, which represents the combine effects of all variables not included in the model.

**3.6.1.6 Coefficients of the Polynomial**

The number of coefficients of the polynomial depends on the number of components and the degree of polynomial the designer wants. The last degree of polynomial possible is equal to the number of components.

Let the number of components be q, and the number of degree of polynomial be m. the least number of components, q in any given mixture is equal to two. Hence

$$2 \leq q \leq \infty \qquad (3.32b)$$

For q = 2, m can be 1

For q = 3, m can be 1, 2 or 3 Or

q = n, m can be 1, 2, 3, …, n

Let the number of coefficient be K; according to Scheffe,

Scheffe (1958) Simon (2003) derived the Eqn of response as;

$$K = \frac{(q+m-1)!}{(q-1)!\,m!} \qquad (3.32c)$$

For a five – Pseudo component mixture used in this work,

q = 5, Let m = 2

$$Thus, K = \frac{(5+2-1)!}{(5-1)!\,2!} = \frac{6!}{4!\,2!} = \frac{6\ x\ 5\ x\ 4!}{4!\ x\ 2\ x\ 1} \qquad (3.32d)$$

$$Hence, \quad K = \frac{6x5}{2} = 15$$

Therefore, the number of coefficients for five Pseudo component mixture with two degree of reaction is 15. This also determines the 15 different mix proportions used for the experiment.

The Equation of response, Y, for six Pseudo component mixture can be given as

$$Y = b_o + \sum b_i X_i + \sum b_{ij} X_i X_j + e \qquad (3.33)$$

Where

$$0 \leq i \leq j \leq 5$$

$i$ and j represent points on the factor space.

Substituting the values of *i* and j gives:

$$Y = b_o + b_1 X_1 + b_2 X_2 + b_3 X_3 + b_4 X_4 + b_5 X_5 + b_{12} X_1 X_2 + b_{13} X_1 X_3 + b_{14} X_1 X_4 + b_{15} X_1 X_5 + b_{23} X_2 X_3 + b_{24} X_2 X_4 + b_{25} X_2 X_5 + b_{34} X_3 X_4 + b_{35} X_3 X_5 + b_{45} X_4 X_5 + b_{11} X_1^2 + b_{22} X_2^2 + b_{33} X_3^2 + b_{44} X_4^2 + b_{55} X_5^2 + e \quad (3.34)$$

Recall, Eqn (3.3)

$$\sum_{i=1}^{q} X_i = 1 \quad (3.35)$$

$$Hence, \sum_{i=1}^{6} X_i = 1 \quad (3.36)$$

This implies that:

$$X_1 + X_2 + X_3 + X_4 + X_5 = 1 \quad (3.37)$$

Multiplying Eqn (3.43) by $b_0$, yields:

$$b_0 X_1 + b_0 X_2 + b_0 X_3 + b_0 X_4 + b_0 X_5 = b_0 (3.38)$$

Multiplying Eqn (3.43) by $X_1$, yields;

$$X_1^2 + X_1 X_2 + X_1 X_3 + X_1 X_4 + X_1 X_5 + = X_1 \quad (3.39)$$

Eqn (3.45) can be transformed to:

$$X_1^2 = X_1 - X_1 X_2 - X_1 X_3 - X_1 X_4 - X_1 X_5 (3.40)$$

Similarly

$$X_2^2 = X_2 - X_1 X_2 - X_2 X_3 - X_2 X_4 - X_2 X_5 (3.41)$$

$$X_3^2 = X_3 - X_1 X_3 - X_2 X_3 - X_3 X_4 - X_3 X_5 (3.42)$$

$$X_4^2 = X_4 - X_1 X_4 - X_2 X_4 - X_3 X_4 - X_4 X_5 (3.43)$$

$$X_5^2 = X_5 - X_1 X_5 - X_2 X_5 - X_3 X_5 - X_4 X_5 (3.44)$$

Substituting Eqn (46) to (51) into Eqn (40), yields.

$$Y = b_0 X_1 + b_0 X_2 + b_0 X_3 + b_0 X_4 + b_0 X_5 + b_1 X_1 + b_2 X_2 + b_3 X_3 + b_4 X_4 + b_5 X_5 + b_{12} X_1 X_2 + b_{13} X_1 X_3 + b_{14} X_1 X_4 + b_{15} X_1 X_5 + b_{23} X_2 X_3 + b_{24} X_2 X_4 + b_{25} X_2 X_5 + b_{34} X_3 X_4 + b_{35} X_3 X_5 + b_{45} X_4 X_5 + b_{11} X_1 - b_{11} X_1 X_2 - b_{11} X_1 X_3 - b_{11} X_1 X_4 - b_{11} X_1 X_5 + b_{22} X_2 - b_{22} X_1 X_2 - b_{22} X_2 X_3 - b_{22} X_2 X_4 - b_{22} X_2 X_5 + b_{33} X_3 - b_{33} X_1 X_3 - b_{33} X_2 X_3 - b_{33} X_3 X_4 - b_{33} X_3 X_5 + b_{44} X_4 - b_{44} X_1 X_4 - b_{44} X_2 X_4 - b_{44} X_3 X_4 - b_{44} X_4 X_5 + b_{55} X_5 - b_{55} X_1 X_5 - b_{55} X_2 X_5 - b_{55} X_3 X_5 - b_{55} X_4 X_5 + e \quad (3.45)$$

Collecting like terms, Eqn (3.45) becomes;

$$Y = X_1(b_o + b_1 + b_{11}) + X_2(b_o + b_2 + b_{22}) + X_3(b_o + b_3 + b_{33}) + X_4(b_o + b_4 + b_{44}) + X_5(b_o + b_5 + b_{55}) + X_1 X_2(b_{12} - b_{11} - b_{22}) + X_1 X_3(b_{13} - b_{11} - b_{33}) + X_1 X_4(b_{14} - b_{11} - b_{44}) + X_1 X_5(b_{15} - b_{11} - b_{55}) + X_2 X_3(b_{23} - b_{22} - b_{33}) + X_2 X_4(b_{24} - b_{22} - b_{44}) + X_2 X_5(b_{25} - b_{22} - b_{55}) + X_3 X_4(b_{34} - b_{33} - b_{44}) + X_3 X_5(b_{35} - b_{33} - b_{55}) + X_4 X_5(b_{45} - b_{44} - b_{55}) + e \quad (3.46)$$

Eqn (3.46) can be expressed in the following form:

$$(b_o + b_i + b_{ii}) + \sum X_i X_j (b_{ij} + b_{ii} + b_{jj}) + e \quad (3.47)$$

Summing up the constant terms in Eqn (3.47) gives:

$$\propto_i = b_o + b_i + b_{ii} (3.48)$$

$$\propto_{ij} = b_{ij} - b_{ii} - b_{jj} \ (3.49)$$

Substituting Eqn (3.55) to (3.56) into Eqn (3.54), yields

$$Y = \sum \propto_i X_i + \sum \propto_{ij} X_i X_j (3.50)$$

Substituting the values in Eqn (3.50) into Eqn (3.46) yields:

$$Y = \propto_1 X_1 + \propto_2 X_2 + \propto_3 X_3 + \propto_4 X_4 + \propto_5 X_5 + \propto_{12} X_1 X_2 + \propto_{13} X_1 X_3 + \propto_{14} X_1 X_4 + \propto_{15} X_1 X_5 + \propto_{23} X_2 X_3 + \propto_{24} X_2 X_4 + \propto_{25} X_2 X_5 + \propto_{34} X_3 X_4 + \propto_{35} X_3 X_5 + \propto_{45} X_4 X_5 + e \quad (3.51)$$

$$Let\ Y = \ddot{Y} + e \quad (3.52)$$

$$\propto_1 = b_o + b_1 + b_{11}, \propto_2 = b_o + b_2 + b_{22}, \propto_3 = b_o + b_3 + b_{33}, \propto_4 = b_o + b_4 + b_{44}, \propto_5 = b_o + b_5 + b_{55}, \propto_{12} = b_{12} - b_{11} - b_{22}, \propto_{13} = b_{13} - b_{11} - b_{33}, \propto_{14} = b_{14} - b_{11} - b_{44}, \propto_{15} = b_{15} - b_{11} - b_{55}, \propto_{23} = b_{23} - b_{22} - b_{33}, \propto_{24} = b_{24} - b_{22} - b_{44}, \propto_{25} = b_{25} - b_{22} - b_{55}, \propto_{34} = b_{34} - b_{33} - b_{44}, \propto_{35} = b_{35} - b_{33} - b_{55}, \propto_{45} = b_{45} - b_{33} - b_{55} \quad (3.54)$$

Substituting the values in Eqn (3.54) into Eqn (3.48) yields:

$$Y = \propto_1 X_1 + \propto_2 X_2 + \propto_3 X_3 + \propto_4 X_4 + \propto_5 X_5 + \propto_{12} X_1 X_2 + \propto_{13} X_1 X_3 + \propto_{14} X_1 X_4 + \propto_{15} X_1 X_5 + \propto_{23} X_2 X_3 + \propto_{24} X_2 X_4 + \propto_{25} X_2 X_5 + \propto_{34} X_3 X_4 + \propto_{35} X_3 X_5 + \propto_{45} X_4 X_5 + e \quad (3.55)$$

$$\ddot{Y} = \sum_{i=1}^{6} \propto_i X_i + \sum_{1 \le i \le j \le 6} \propto_{ij} X_i X_{ij} \, (3.56a)$$

Eqn (3.60a) is the response of the pure component "$i$" and the binary component "ij"

If the response function is represented by y, the response function for the pure component and that for the binary mixture components will be $y_i$ and $y_{ij}$ respectively.

$$Y_i = \sum_{i=1}^{6} \propto_i X_i \, (3.57a)$$

$$y_i = \sum_{i=1}^{6} \propto_i X_i + \sum_{1 \le i \le j \le 6} \propto_{ij} X_i X_{ij} \, (3.57b)$$

Where e = standard error, and

$$\ddot{Y} = \propto_1 X_1 + \propto_2 X_2 + \propto_3 X_3 + \propto_4 X_4 + \propto_5 X_5 + \propto_{12} X_1 X_2 + \propto_{13} X_1 X_3 + \propto_{14} X_1 X_4 + \propto_{15} X_1 X_5 + \propto_{23} X_2 X_3 + \propto_{24} X_2 X_4 + \propto_{25} X_2 X_5 + \propto_{34} X_3 X_4 + \propto_{35} X_3 X_5 + \propto_{45} X_4 X_5 + e \quad (3.53a)$$

From Eqn (3.48) and (3.49), the constant term in Eqn (3.46) can be written out as follows:

If the response at $i^{th}$ point on the factor space is $y_i$, then at point 1, component $X_1 = 1$ and components $X_2$, $X_3$, $X_4$, $X_5$, $X_s$ are all equal to zero at $X_1 = 1$ Eqn (3.57a) becomes

$$y_1 = \propto_1 \, (3.58)$$

Substituting $X_2 = 1$ and $X_1 = X_3 = X_4 = X_5 = X_6 = 0$ Eqn (3.61a) becomes;

$$y_2 = \propto_2 \, (3.59)$$

Similarly,

$$y_3 = \propto_3 \, (3.60)$$

$$y_4 = \propto_4 \, (3.61)$$

$$y_5 = \propto_5 \, (3.62)$$

$$y_6 = \propto_6 \, (3.63)$$

Eqns (3.58) to (3.63) can be expressed in the form

$$y_i = \propto_i \, (3.64)$$

For point 12, that is the mid-point of the borderlines connecting points 1 and 2 of the factor space, component $X_1 = \frac{1}{2}$; $X_2 = \frac{1}{2}$ and $X_3 = X_4 = X_5 = 0$. The response at this point is $y_{12}$.

In Eqn (3.61b), the response, $y_{12}$ becomes;

$$y_{12} = \frac{1}{2} \propto_1 + \frac{1}{2} \propto_2 + \left( \propto_{12} . \frac{1}{2} \, . \frac{1}{2} \right)$$

$$y_{12} = \frac{1}{2} \propto_1 + \frac{1}{2} \propto_2 + \frac{1}{4} \propto_{12} \, (3.65)$$

Similarly

$$y_{13} = \frac{1}{2} \propto_1 + \frac{1}{2} \propto_3 + \frac{1}{4} \propto_{13} \ (3.66)$$

$$y_{14} = \frac{1}{2} \propto_1 + \frac{1}{2} \propto_4 + \frac{1}{4} \propto_{14} \ (3.67)$$

$$y_{15} = \frac{1}{2} \propto_1 + \frac{1}{2} \propto_5 + \frac{1}{4} \propto_{15} \ (3.68)$$

$$y_{23} = \frac{1}{2} \propto_2 + \frac{1}{2} \propto_3 + \frac{1}{4} \propto_{23} \ (3.69)$$

$$y_{24} = \frac{1}{2} \propto_2 + \frac{1}{2} \propto_4 + \frac{1}{4} \propto_{24} \ (3.70)$$

$$y_{25} = \frac{1}{2} \propto_2 + \frac{1}{2} \propto_5 + \frac{1}{4} \propto_{25} \ (3.71)$$

$$y_{34} = \frac{1}{2} \propto_3 + \frac{1}{2} \propto_4 + \frac{1}{4} (3.72)$$

$$y_{35} = \frac{1}{2} \propto_3 + \frac{1}{2} \propto_5 + \frac{1}{4} \propto_{35} \ (3.73)$$

$$y_{45} = \frac{1}{2} \propto_4 + \frac{1}{2} \propto_5 + \frac{1}{4} \propto_{45} \ (3.74)$$

Eqns (3.69) – (3.74) can be written in the form;

$$y_{ij} = \frac{1}{2} \propto_i + \frac{1}{2} \propto_j + \frac{1}{4} \propto_{ij} \ (3.75)$$

Rearranging Eqns (3.59) and (3.75), gives

$$\propto_i = \ y_i (3.76)$$

$$\propto_{ij} = \ 4y_{ij} \ - 2 \propto_i - 2 \propto_i \ (3.77)$$

$$where \ \propto_i = y_i \ and \ \propto_j = 2_j (3.78)$$

Substituting Eqn (3.74) into Eqn (3.72), yields

$$\propto_{ij} = \ 4y_{ij} \ - 2y_i - 2y_i (3.79)$$

Substituting Eqns (3.77) and (3.79) into Eqn (3.52), yields;

$$\begin{aligned} Y = \ & y_1 X_1 + y_2 X_2 + y_3 X_3 + y_4 X_4 + y_5 X_5 \\ & + (4y_{12} - 2X_1 - 2X_2) X_1 X_2 \\ & + (4y_{13} - 2X_1 - 2X_3) X_1 X_3 \\ & + (4y_{14} - 2X_1 - 2X_4) X_1 X_4 \\ & + (4y_{15} - 2X_1 - 2X_5) X_1 X_5 \\ & + (4y_{23} - 2X_2 - 2X_3) X_2 X_3 \\ & + (4y_{24} - 2X_2 - 2X_4) X_2 X_4 \\ & + (4y_{25} - 2X_2 - 2X_5) X_2 X_5 \\ & + (4y_{34} - 2X_3 - 2X_4) X_3 X_4 \\ & + (4y_{35} - 2X_3 - 2X_5) X_3 X_5 \\ & + (4y_{45} - 2X_4 - 2X_5) X_4 X_5 \\ & + e \qquad (3.80) \end{aligned}$$

Expanding Eqn (3.80) and rearranging gives;

$$\begin{aligned} Y \\ = \ & y_1 X_1 - 2y_1 X_1 X_2 - 2y_1 X_1 X_3 - 2y_1 X_1 X_4 \\ & - 2y_1 X_1 X_5 + y_2 X_2 - 2y_2 X_1 X_2 - 2y_2 X_2 X_3 \\ & - 2y_2 X_2 X_4 - 2y_2 X_2 X_5 + y_3 X_3 - 2y_3 X_1 X_3 \\ & - 2y_3 X_2 X_3 - 2y_3 X_3 X_4 - 2y_3 X_3 X_5 + y_4 X_4 \\ & - 2y_4 X_1 X_4 - 2y_4 X_2 X_4 - 2y_4 X_3 X_4 \\ & - 2y_4 X_4 X_5 + y_5 X_5 - 2y_5 X_1 X_5 - 2y_5 X_2 X_5 \\ & - 2y_5 X_3 X_5 - 2y_5 X_4 X_5 + 4y_{12} X_1 X_2 \\ & + 4y_{13} X_1 X_3 + 4y_{14} X_1 X_4 + 4y_{15} X_1 X_5 \\ & + 4y_{23} X_2 X_3 + 4y_{24} X_2 X_4 + 4y_{25} X_2 X_5 \\ & + 4y_{34} X_3 X_4 + 4y_{35} X_3 X_5 + 4y_{45} X_4 X_5 \\ & + e \qquad (3.81) \end{aligned}$$

Factorizing Eqn (3.81), gives

$$\begin{aligned} Y = \ & y_1 X_1 (1 - 2X_2 - 2X_3 - 2X_4 - 2X_5) \\ & + y_2 X_2 (1 - 2X_1 - 2X_3 \\ & - 2X_4 - 2X_5) \\ & + y_3 X_3 (1 - 2X_1 - 2X_2 \\ & - 2X_4 - 2X_5) \\ & + y_4 X_4 (1 - 2X_1 - 2X_2 \\ & - 2X_3 - 2X_4) \\ & + y_5 X_5 (1 - 2X_1 - 2X_2 \\ & - 2X_3 - 2X_4) + 4y_{12} X_1 X_2 \\ & + 4y_{13} X_1 X_3 + 4y_{14} X_1 X_4 \\ & + 4y_{15} X_1 X_5 + 4y_{23} X_2 X_3 \\ & + 4y_{24} X_2 X_4 + 4y_{25} X_2 X_5 \\ & + 4y_{26} X_2 X_6 + 4y_{34} X_3 X_4 \\ & + 4y_{35} X_3 X_5 + 4y_{45} X_4 X_5 \\ & + e \qquad (3.82) \end{aligned}$$

Recall that in Eqn (3.43);

$$X_1 + X_2 + X_3 + X_4 + X_5 = 1$$

Multiplying Eqn (3.43) by 2 gives

$$2X_1 + 2X_2 + 2X_3 + 2X_4 + 2X_5 = 2 \qquad (3.83)$$

Subtracting 1 from both sides of Eqn (3.83), gives

$$2X_1 + 2X_2 + 2X_3 + 2X_4 + 2X_5 - 1 = 1 \quad (3.84)$$

Eqn (3.84) can be expressed as:

$$2X_1 - 1 = 1 - 2X_2 - 2X_3 - 2X_4 - 2X_5 (3.85)$$

Similarly,

$$2X_2 - 1 = 1 - 2X_1 - 2X_3 - 2X_4 - 2X_5 (3.86)$$

$$2X_3 - 1 = 1 - 2X_1 - 2X_2 - 2X_4 - 2X_5 (3.87)$$

$$2X_4 - 1 = 1 - 2X_1 - 2X_3 - 2X_4 - 2X_5 (3.88)$$

$$2X_5 - 1 = 1 - 2X_1 - 2X_2 - 2X_3 - 2X_4 (3.89)$$

Substituting Eqns (3.83) to (3.89) into Eqn (3.81), yield

$$Y = X_1(2X_1 - 1)y_1 + X_2(2X_2 - 1)y_2 + X_3(2X_3 - 1)y_3 + X_4(2X_4 - 1)y_4 + X_5(2X_5 - 1)y_5 + 4y_{12} X_1 X_2 + 4y_{13} X_1 X_3 + 4y_{14} X_1 X_4 + 4y_{15} X_1 X_5 + 4y_{23} X_2 X_3 + 4y_{24} X_2 X_4 + 4y_{25} X_2 X_5 + 4y_{26} X_2 X_6 + 4y_{34} X_3 X_4 + 4y_{35} X_3 X_5 + 4y_{45} X_4 X_5 + e \qquad (3.90)$$

Eqn (3.90) is the mixture design mode for the optimization of a concrete mixture consisting of five components. The term, $y_i$ and $y_{ij}$ responses (representing compressive strength) at the point *i*and *ij*. These responses are determined by carrying out laboratory tests.

**Control Points**

Another set of fifiteen mix proportions are required to confirm the adequacy of the model of Eqn (3.90). The set of mixture proportions are called control mixture proportions. Therefore, twenty-one control points will be used. They are $C_1$, $C_2$, $C_3$, $C_4$, $C_5$, $C_{12}$, $C_{13}$, $C_{14}$, $C_{15}$, $C_{23}$, $C_{24}$, $C_{25}$, $C_{34}$, $C_{35}$, and $C_{45}$,

The mass constituent of the ingredients of concrete for both trial and control mixes are as shown in Tables 3.5 and 3.6 respectively.

**Table3.6: Mass of Constituents of Concrete of Control Mixes (kg)**

| C | Water | Cement | SCBA | Sand | Granite |
|---|---|---|---|---|---|
| 1 | 1.95 | 3.435 | 0.375 | 9.405 | 13.485 |
| 2 | 1.905 | 3.3 | 0.585 | 8.895 | 13.98 |
| 3 | 1.86 | 3.18 | 0.6375 | 8.4 | 14.49 |
| 4 | 1.905 | 3.345 | 0.477 | 9.36 | 13.56 |
| 5 | 1.845 | 3.195 | 0.6225 | 8.835 | 14.13 |
| 12 | 1.86 | 3.24 | 0.573 | 8.97 | 10.125 |
| 13 | 1.89 | 3.3 | 0.525 | 9.165 | 13.74 |
| 14 | 1.785 | 3.15 | 0.669 | 9.54 | 13.365 |
| 15 | 1.95 | 3.405 | 0.42 | 9.00 | 13.905 |
| 23 | 1.935 | 3.36 | 0.465 | 8.865 | 14.055 |
| 24 | 1.92 | 3.3 | 0.495 | 8.7 | 14.205 |
| 25 | 1.8 | 3.285 | 0.54 | 8.55 | 12.885 |
| 34 | 1.92 | 3.27 | 0.54 | 8.325 | 14.58 |
| 35 | 1.92 | 3.27 | 0.555 | 8.28 | 14.625 |

| | | | | | |
|---|---|---|---|---|---|
| 45 | 1.965 | 3.405 | 0.42 | 9.045 | 14.085 |

**4.0 Compressive Test on Sugar Cane Bagasse ash – Cement Concrete**

This test was conducted on concrete cubes to determine compressive strength of each replicate cube after 28days of curing (28$^{th}$days strength). The compressive strength of each replicate cube was calculated using *equation 4.1*and the mean compressive strength was calculated using *equation 4.2*, the equations are stated below

Compressive Strength =

$$\frac{Force\ applied\ \ at\ rupture}{Area\ on\ which\ the\ force\ \ is\ applied} = \frac{F}{A} \quad 4.1$$

$$Meancompressivestrength = \frac{compressivestrengthofreplicate1 + compressivestrengthofreplicate2 + compressivestrengthofreplicate3}{3} \quad (4.2)$$

The "F" value is read from the compressive machine when cube crushed A = 150 x 150mm$^2$(since cube used for the work is a 150 x 150 x150 cube). The 28$^{th}$day compressive strength of each mix is presented in the table4.1

**Table 4.1 Compressive Strength Test Results of 28$^{th}$ Day of Concrete Cube**

| S/No | Point of observation | Replicate 1 (N/mm$^2$) | Replicate 2 (N/mm$^2$) | Replicate 3 (N/mm$^2$) | Mean Compressive Strength (N/mm$^2$) |
|---|---|---|---|---|---|
| 1 | 1 | 23.22 | 20.93 | 21.51 | 21.89 |
| 2 | 2 | 28.71 | 29.16 | 30.58 | 29.48 |
| 3 | 3 | 22.24 | 23.16 | 23.22 | 21.54 |
| 4 | 4 | 15.44 | 16.13 | 15.13 | 15.57 |
| 5 | 5 | 15.56 | 14.49 | 15.47 | 15.17 |
| 6 | 12 | 24.09 | 24.60 | 25.87 | 24.85 |
| 7 | 13 | 21.36 | 19.40 | 21.93 | 20.90 |
| 8 | 14 | 20.47 | 16.02 | 17.71 | 18.07 |
| 9 | 15 | 21.80 | 25.44 | 25.89 | 24.38 |
| 10 | 23 | 26.33 | 23.89 | 22.78 | 24.33 |
| 11 | 24 | 13.89 | 13.80 | 13.00 | 13.56 |
| 12 | 25 | 13.84 | 15.22 | 13.40 | 14.15 |
| 13 | 34 | 15.22 | 20.20 | 15.31 | 16.91 |
| 14 | 35 | 21.18 | 17.36 | 21.18 | 18.91 |
| 15 | 45 | 16.82 | 13.44 | 13.80 | 14.69 |
| 16 | $C_1$ | 22.95 | 23.59 | 21.80 | 22.78 |
| 17 | $C_2$ | 18.31 | 18.05 | 18.33 | 18.23 |
| 18 | $C_3$ | 18.79 | 19.29 | 19.67 | 19.25 |
| 19 | $C_4$ | 19.40 | 18.78 | 17.77 | 18.65 |
| 20 | $C_5$ | 19.04 | 19.67 | 19.25 | 19.32 |
| 21 | $C_6$ | 18.96 | 16.38 | 17.28 | 17.54 |
| 22 | $C_7$ | 20.35 | 22.16 | 20.55 | 21.02 |
| 23 | $C_8$ | 13.44 | 15.47 | 13.72 | 14.21 |
| 24 | $C_9$ | 18.70 | 20.85 | 18.20 | 19.25 |
| 25 | $C_{10}$ | 22.45 | 22.08 | 21.53 | 22.02 |
| 26 | $C_{11}$ | 20.18 | 19.67 | 20.18 | 20.01 |

| | | | | | |
|---|---|---|---|---|---|
| 27 | $C_{12}$ | 20.33 | 19.50 | 20.80 | 20.21 |
| 28 | $C_{13}$ | 21.33 | 20.10 | 22.20 | 21.21 |
| 29 | $C_{14}$ | 21.50 | 20.32 | 21.27 | 21.03 |
| 30 | $C_{15}$ | 18.58 | 17.88 | 18.95 | 18.47 |

**Table 4.4 Comparison of the Compressive Strength Obtained fromthe Model and the Experiment**

| Points | Experimental | Computed Compressive | A | B |
|---|---|---|---|---|
| of | Compressive Strength | Strength | | (%) |
| observation | (N/mm2) | (N/mm2) | | |
| | | Scheffe's Model | | |
| 1 | 21.89 | 21.89 | 0 | 0 |
| 2 | 29.48 | 29.48 | 0 | 0 |
| 3 | 21.54 | 21.54 | 0 | 0 |
| 4 | 15.57 | 15.57 | 0 | 0 |
| 5 | 15.17 | 15.17 | 0 | 0 |
| 12 | 24.85 | 24.85 | 0 | 0 |
| 13 | 20.9 | 20.9 | 0 | 0 |
| 14 | 18.07 | 18.07 | 0 | 0 |
| 15 | 24.38 | 24.38 | 0 | 0 |
| 23 | 24.33 | 24.33 | 0 | 0 |
| 24 | 13.56 | 13.56 | 0 | 0 |
| 25 | 14.15 | 14.15 | 0 | 0 |
| 34 | 16.91 | 16.91 | 0 | 0 |
| 35 | 18.91 | 18.91 | 0 | 0 |
| 45 | 14.69 | 14.69 | 0 | 0 |
| C1 | 22.78 | 22.98 | -0.2 | -0.87413 |
| | | | | |
| C2 | 18.23 | 18.224 | 0.01 | 0.05487 |
| | | | | |
| C3 | 19.25 | 19.491 | -0.24 | -1.23903 |
| | | | | |
| C4 | 18.65 | 18.595 | 0.05 | 0.268456 |
| | | | | |
| C5 | 19.32 | 19.194 | 0.13 | 0.675149 |

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| | | | | |
|---|---|---|---|---|
| C6 | 17.54 | 17.161 | 0.38 | 2.190202 |
| | | | | |
| C7 | 21.02 | 20.87 | 0.15 | 0.716161 |
| | | | | |
| C8 | 14.21 | 14.418 | -0.21 | -1.46699 |
| | | | | |
| C9 | 19.25 | 19.488 | -0.24 | -1.23903 |
| | | | | |
| C10 | 22.02 | 21.93 | 0.09 | 0.409556 |
| | | | | |
| C11 | 20.01 | 19.342 | 0.67 | 3.405337 |
| | | | | |
| C12 | 20.21 | 20.329 | -0.12 | -0.59201 |
| | | | | |
| C13 | 21.21 | 20.823 | 0.39 | 1.855817 |
| | | | | |
| C14 | 21.03 | 21.245 | -0.22 | -1.04068 |
| | | | | |
| C15 | 18.47 | 18.467 | 0.03 | 0.01 |

A= Difference between results obtained from Experimental investigation and Scheffe's Model

B= Percentage difference between results obtained from Experimental investigation and Scheffe's Model

$$\text{Percentage Difference} = \frac{\text{Difference of x and y}}{\text{Average of x and y}} \; x \; 100\% \qquad (4.3)$$

**Determination of Compressive Strength from Scheffe's Simplex Model**

The Scheffe's Simplex Model used in writing the computer program is obtained by substituting the values of the compressive strength results ($Y_i$)from table 4.6 into Scheffe's model given in *equation (3.81)*

Substituting these values gives *Equation 4.5*

$$Y = 21.89X_1(2X_1\text{-}1) + 29.48X_2(2X_2\text{-}1) + 21.54X_3(2X_3\text{-}1) + 15.57X_4(2X_4\text{-}1) + 15.17X_5(2X_5\text{-}1) + 99.4X_1X_2 + 83.6X_1X_3 + 72.28X_1X_4 + 97.52X_1X_5 + 97.32X_2X_3 + 54.24X_2X_4 + 56.6X_2X_5 + 67.64X_3X_4 + 75.64X_3X_5 + 58.76X_4X_5 \qquad (4.4)$$

*Equation 4.5 is the Scheffe's Simplex Design model for the optimization of the compressive strength of Sugar Cane Bagasse ash Cement Concrete*

**Test of Adequacy of Scheffe's Model**

T- Statistic tests will be used to testing the adequacy of Scheffe's model developed, it is expected that the results of the model will be about 95% accurate

**Table 4.5 T – statistical test computation for Scheffe's Simplex Model**

| SN | YE | YM | Di =YM-YE | DA – Di | (DA - Di)2 |
|---|---|---|---|---|---|
| C1 | 22.78 | 22.98 | -0.2 | -0.212 | 0.0449 |
| C2 | 18.23 | 18.22 | 0.01 | -0.422 | 0.1781 |
| C3 | 19.25 | 19.49 | -0.24 | -0.172 | 0.0296 |
| C4 | 18.65 | 18.6 | 0.05 | -0.462 | 0.2134 |
| C5 | 19.32 | 19.2 | 0.12 | -0.532 | 0.283 |
| C6 | 17.54 | 23.24 | -5.7 | 5.288 | 27.9629 |
| C7 | 21.02 | 14.79 | 6.23 | -6.642 | 44.1162 |
| C8 | 14.21 | 15.42 | -1.21 | 0.798 | 0.6368 |
| C9 | 19.25 | 19.45 | -0.2 | -0.212 | 0.0449 |

| | | | | | |
|---|---|---|---|---|---|
| C10 | 22.02 | 18.04 | 3.98 | -4.392 | 19.2897 |
| C11 | 20.01 | 23.24 | -3.23 | 2.818 | 7.9411 |
| C12 | 20.21 | 20.33 | -0.12 | -0.292 | 0.0853 |
| C13 | 21.21 | 21.8 | -0.59 | 0.178 | 0.0317 |
| C14 | 21.03 | 21.25 | -0.22 | -0.192 | 0.0369 |
| C15 | 18.47 | 23.33 | -4.86 | 4.448 | 19.7847 |
| | | S Di = | -6.18 | S (DA - Di)$^2$ = | 120.6792 |
| | | DA = S Di / N = | -0.412 | S2 = S (DA Di)$^2$/(N-1) = | 8.6199 |
| | | | | S = Ö S2 = | 2.936 |
| | | | | **T = DA*(N)^0.5/S =** | -0.5435 |

$T_{CALCULATED}$ =**0.5435**

5 % Significance for Two-Tailed Test = 2.5 %
1 - 2.5% = 0.975

The value of Allowable Total Variation In T-Test is obtained from standard T – statistic table Allowable Total Variation In T- Test = $T_{(0.975,\ N-1)}$ = $T_{(0.975,\ 14)}$ = **2.14**

The value of $T_{calculated}$(0.5435) is below the allowable total variation (2.14), the null hypothesis that "there is no significant difference between the experimental and the model expected results" is accepted. This implies that Scheffe Simplex Model is adequate

### 5.0 Conclusion

From this research work it can be concluded that;

i. The result of the compressive strength test showed that the strength of the Sugar cane Bagasse ash cement- concrete was highest at 10% replacement of Cement with Sugar Cane Bagasse Ash. The result of these tests shows the feasibility of using SCBA as partial replacement for cement. It also makes the colour of the concrete to be darker than ordinary conventional concrete.

ii. A mathematical model was developed using Scheffe's Simplex Model which was used to predict the compressive strength give a mix ratio and a mix ratio given a compressive strength.

iii. The student t-test and the fisher-statistical test were used to check the adequacy of the model and model was found to be adequate at 95% confidence level.

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